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Appendix C
Giroud and Bonaparte (1989b)

Leakage through Liners Constructed with Geomembranes—Part II. Composite Liners*

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3 LEAKAGE THROUGH COMPOSITE LINERS

3.1 Introduction

3.1.1 Scope of the section

As indicated in Section 1.2.4, two types of liners are considered: geomembrane liners (i.e. geomembrane alone) and composite liners (i.e. liners comprised of a geomembrane associated with a layer of low-permeability soil). Section 2 discussed leakage through geomembrane liners. Leakage through composite liners is discussed in this section.

3.1.2 Leakage mechanisms

Leakage through a composite liner. A composite liner is comprised of a geomembrane upper component and a low-permeability soil layer lower component. Therefore, leakage migrates first through the geomembrane component and, then, through the soil component.

Leakage through the geomembrane component of a composite liner. As indicated in Section 2.1.2, there are two mechanisms by which leakage can migrate through a geomembrane:

- permeation through the geomembrane (i.e. flow through a geomembrane that has no defects); and
- flow through geomembrane defects such as holes or pinholes.

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Leakage rates due to permeation through the geomembrane component of a composite liner should not be significantly affected by the presence of the underlying low-permeability soil layer because even a soil with a very low permeability is still very permeable as compared to a geomembrane without defects. Therefore, data and discussions from Section 2.2 on permeation through a geomembrane without defects are applicable to composite liners, and Section 3 will be devoted to leakage through composite liners due to a hole in the geomembrane.

Leakage through a composite liner due to a geomembrane hole. The mechanism of leakage through a composite liner when there is a hole in the geomembrane is as follows: the fluid (i.e. liquid or vapor) first migrates through the geomembrane hole; the fluid may then travel laterally some distance in the space, if any, between the geomembrane and the low-permeability soil; finally, the fluid migrates into and eventually through the low-permeability soil. This leakage mechanism is applicable to both liquids and gases. However, in the remainder of this section, consideration will be limited to the leakage of liquids through composite liners.

There may be no space between the geomembrane component and the soil component of a composite liner if the geomembrane is sprayed directly onto the low-permeability soil layer. This technique is not very often used, and, in the more usual case of a geomembrane manufactured in a plant, there will be some space between the geomembrane component and the soil component of a composite liner in almost all applications because:

- the geomembrane has wrinkles (note that geomembrane wrinkles may exist even under very high pressures as shown by Stone¹⁴);
- there are clods or irregularities at the underlying soil surface; and/or
- even if the underlying soil surface is apparently smooth, the geomembrane bridges small spaces between soil particles.

Test results discussed in Section 3.3 seem to indicate that some lateral flow almost always occurs between the geomembrane and the underlying soil, even under good laboratory test conditions when the geomembrane is placed as flat as possible on a soil layer that has a smooth surface.

Influence of overlying material. As discussed in Section 2.3.3, leakage through a hole in a geomembrane liner underlain by a pervious material can be significantly impeded by the overlying material. The case of a composite liner is completely different: the flow is essentially governed by the low-permeability soil underlying the geomembrane, i.e. the head loss at the geomembrane hole is negligible compared to the head loss in the low-permeability soil. Therefore, if the material overlying the geomembrane is more permeable than the low-permeability soil component of the

composite liner (which is practically always the case), no significant head loss will take place in the overlying material. Therefore, the presence of the overlying material will not significantly affect the leakage rate, unless fine particles migrating from the overlying material clog the geomembrane hole and/or the space between the geomembrane and low-permeability soil.

From the above discussion, it may be concluded that, for all practical purposes, the rate of leakage through a composite liner is independent of the overlying material.

3.1.3 Organization of the section

Because the leakage mechanisms presented above are complex, it is appropriate to consider different and complementary approaches. Accordingly, the next two sections (3.2 and 3.3) are devoted to analytical studies and laboratory model tests, respectively. Lastly, practical conclusions regarding the evaluation of leakage rates through composite liners are presented in Section 3.4.

3.2 Analytical studies

3.2.1 Introduction

As indicated in Section 3.1.2, liquid that has passed through a hole in the geomembrane component of a composite liner may flow laterally some distance between the geomembrane and the low-permeability soil before migrating into the low-permeability soil. This 'interface flow' is possible only if there is a space between the geomembrane and the low-permeability soil. There is no interface flow if the geomembrane and the soil are in perfect contact (which is an ideal case that is extremely difficult to achieve in practice).

Accordingly, two types of analytical studies are presented hereafter:

- analytical studies assuming that there is perfect contact between the geomembrane and the low-permeability soil, and, consequently, liquid does not flow laterally between the geomembrane and the low-permeability soil; and
- analytical studies assuming that liquid flows laterally between the geomembrane and the low-permeability soil before migrating into the low-permeability soil.

In all cases, analyses presented in this section are based on the assumption of steady-state saturated flow conditions.

Three-dimensional analyses of this complex problem are difficult and it

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is appropriate to start with two-dimensional analyses. Accordingly, the remainder of Section 3.2 is organized as follows:

- Two-dimensional analyses assuming perfect contact (Section 3.2.2).
- Three-dimensional analyses assuming perfect contact (Section 3.2.3).
- Three-dimensional analyses with interface flow (Section 3.2.4).

3.2.2 Two-dimensional analyses assuming perfect contact

Faure¹⁸ has made an extensive two-dimensional theoretical analysis (using numerical methods) of the leakage through a composite liner due to a hole in the geomembrane, assuming perfect contact between the geomembrane and the underlying low-permeability soil. Most of Section 3.2.2 is a summary of Faure's work. A two-dimensional study was also made by Sherard¹⁹ who traced flow nets by trial and error for a limited number of cases.

Assumptions. The two-dimensional case discussed in this section is defined as follows (Fig. 5(a)):

- The hole in the geomembrane is a slot with a width b and an infinite length in the direction perpendicular to the considered cross-section.
- The depth of liquid on top of the geomembrane is h_w .
- The thickness of the low-permeability soil layer beneath the geomembrane is H_s and its hydraulic conductivity is k_s .

Description of the flow. Both Faure and Sherard have shown that for two-dimensional flow (Fig. 5(b)):

- there is horizontal flow in the soil *along* a portion of the interface (although there is no flow *between* the geomembrane and the soil since perfect contact is assumed); and
- there is a phreatic surface beyond which the soil is not saturated.

These qualitative characteristics of two-dimensional flow are certainly also applicable to the three-dimensional case (circular hole). Examples of equipotential lines for the two-dimensional case are given in Fig. 6 and a chart giving the location of the phreatic surface in the two-dimensional case is presented in Fig. 7.

Leakage rate. A chart giving the leakage rate when the geomembrane and the underlying soil are in perfect contact was given by Faure^{18,20} for the two-dimensional case (Fig. 8). The results given by Sherard¹⁹ for a limited number of cases are consistent with Faure's. Faure's chart (Fig. 8) is used with the following equation:

$$Q^* = C_F k_s (H_s + h_w) \quad (23)$$

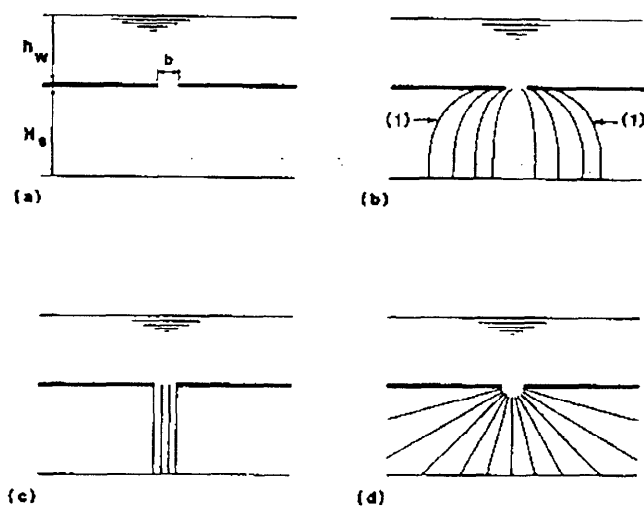


Fig. 5. Two-dimensional flow through a composite liner due to a geomembrane defect, assuming perfect contact between geomembrane and soil layer: (a) definition of the two-dimensional case considered; (b) correct solution; (c) vertical flow giving a lower bound of the leakage rate; (d) radial flow giving an upper bound of the leakage rate. As demonstrated by Faure:¹⁸ (i) the flow is limited laterally by a phreatic surface (i.e. there is no flow beyond surfaces (1) in Fig. 5(b)); and (ii) there is flow in the soil along a portion of the geomembrane-soil interface (although there is no flow between the geomembrane and the soil because there is no space between the geomembrane and the soil in the case of perfect contact).

where: Q^* = leakage rate per unit length in the direction perpendicular to the figure; C_F = dimensionless coefficient given by the chart as a function of b/H_s and h_w/H_s ; b = width of the slot; k_s = hydraulic conductivity of the low-permeability soil underlying the geomembrane; H_s = thickness of the low-permeability soil underlying the geomembrane; and h_w = depth of liquid on top of the geomembrane. Basic SI units are: Q^* (m^2/s), b (m), k_s (m/s), H_s (m), and h_w (m).

Lower bound solution. If the flow is assumed to be vertical (Fig. 5(c)), the leakage rate is given by a close-form solution obtained by applying Darcy's equation to a rectangular domain:

$$Q^* = k_s b (h_w + H_s) / H_s \quad (24)$$

where the notation is the same as above.

A comparison of Figs 5(b) and 5(c) shows that the area of flow for the case of vertical flow (Fig. 5(c)) is only a fraction of the area of flow for the correct solution (Fig. 5(b)). Therefore, it may be concluded that the

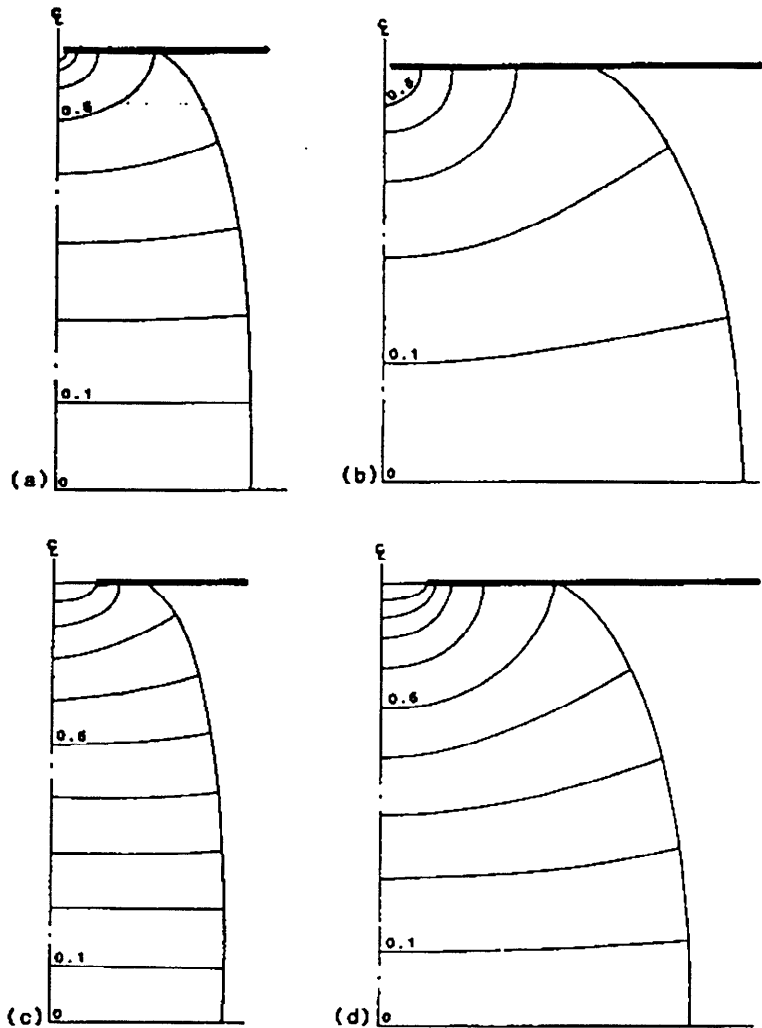


Fig. 6. Typical equipotential lines for leakage through a composite liner due to a geomembrane hole. These equipotential lines result from a two-dimensional study assuming that the geomembrane and the underlying soil are in perfect contact (see case (b) in Fig. 5) (from Faure).¹⁸ The cases shown above are: (a) $b/H_s = 0.02$ and $h_w/H_s = 1$; (b) $b/H_s = 0.02$ and $h_w/H_s = 3$; (c) $b/H_s = 0.2$ and $h_w/H_s = 1/3$; and (d) $b/H_s = 0.2$ and $h_w/H_s = 1$. Notation: b = width of infinitely long hole (slot) in the geomembrane; h_w = depth of liquid on top of the geomembrane; and H_s = thickness of the low-permeability soil layer underlying the geomembrane. Note that there is no flow beyond the phreatic surface.

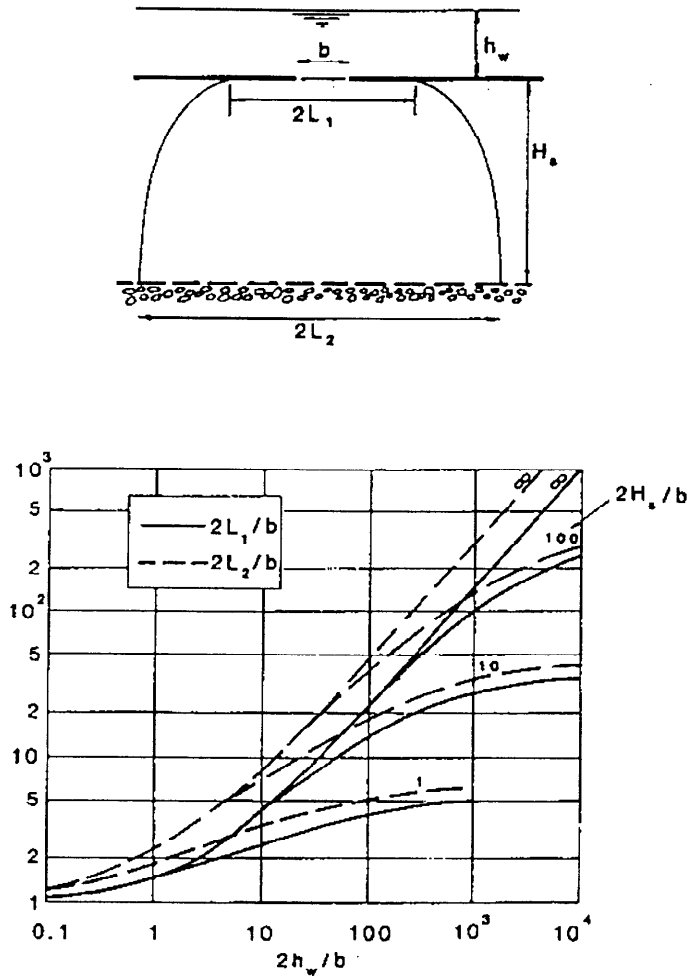


Fig. 7. Lateral extent of the phreatic surface limiting flow in the soil layer due to a hole in the geomembrane (from Faure).¹⁸ This chart is related to the two-dimensional case (the hole is a slot of width b) and perfect contact between the geomembrane and the soil layer. The chart shows that, when the water depth becomes very large, L_1/b and L_2/b become very large, whereas L_1/H_s and L_2/H_s reach a limiting value of the order of 5. In other words, the lateral extent of the saturated zone can be large compared to the hole size, but not more than a few times the thickness of the soil layer.

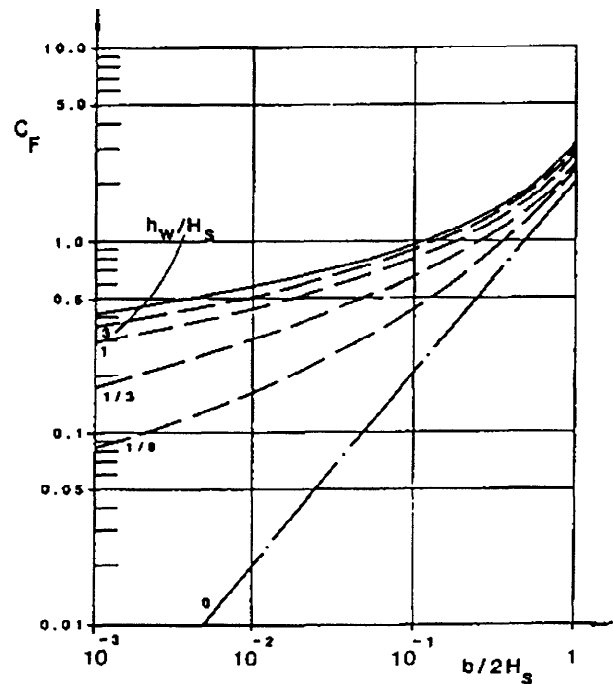


Fig. 8. Chart for calculation of leakage rate due to geomembrane hole when the geomembrane is in perfect contact with a low-permeability soil. This chart gives the dimensionless coefficient C_F to be used in eqn (23) which gives the leakage rate through a composite liner due to a slot in the geomembrane (two-dimensional case). This chart was established by Faure.^{18,20} The coefficient C_F can also be used in eqn (31) to make an approximate evaluation of the leakage rate through a composite liner due to a circular hole in the geomembrane (three-dimensional case). Notation: h_w = depth of liquid on the geomembrane; b = width of the slot (to be replaced by the diameter d of a circular hole when the chart is used for the three-dimensional case); and H_s = thickness of soil layer. Comments: (i) if $h_w/H_s = 0$, the flow is vertical (Fig. 5(c)); and (ii) C_F has the same value for all values of h_w/H_s greater than 5.

leakage rate obtained using eqn (24) is a lower bound solution for the leakage rate through the composite liner when the geomembrane and the underlying soil are in perfect contact. This lower bound solution gives a good approximation of the leakage rate if the ratio between the width of the geomembrane hole and the thickness of the low-permeability soil is large, which is rare.

Upper bound solution. If the flow is assumed to be radial (Fig. 5(d)), the leakage rate is given by a close-form solution obtained by integrating Darcy's equation for a circular domain:

$$Q^* = \pi k_s (h_w + H_s) / \ln(2H_s/b) \quad (25)$$

where the notation is the same as for eqn (23).

In some cases, eqn (25) leads to absurd results, such as flow rates increasing as soil thickness increases. However, this case is useful because Faure showed that it provides an upper bound for the leakage rate through the composite liner when the geomembrane and the underlying soil are in perfect contact. Also the fact that the leakage rate in the case of radial flow is expressed by a close-form solution for the three-dimensional case (circular hole) as well as for the two-dimensional case provides a convenient way to compare the three-dimensional case with the two-dimensional case. Any relationship between the two- and three-dimensional cases is useful because the three-dimensional case is very difficult to analyse.

Approximate solution. The upper bound solution provided by the radial flow equation (eqn (25)) is excessively high in many cases. In addition, for H_s/h_w larger than a certain value, the flow rate increases with increasing soil thickness, as shown in Fig. 9 (case c). Since the leakage rate cannot increase if the thickness of the soil layer increases while all other variables remain constant, the upper bound cannot be used as an approximation for the leakage rate when H_s/h_w is large.

Equation (25) can be arbitrarily transformed by replacing $h_w + H_s$ by h_w , which gives:

$$Q^* = \pi k_s h_w / \ln(2H_s/b) \quad (26)$$

As it turns out, this equation can be used for large values of H_s/h_w where it gives values (case (c') in Fig. 9) less than, but not too far from, the values calculated using eqn (23).

Usefulness of the two-dimensional case. The above considerations regarding boundaries and approximations related to the two-dimensional case will provide useful guidance for an approximate evaluation of the leakage rate in the three-dimensional case (circular hole) where no numerical solution similar to eqn (23) is available.

3.2.3 Three-dimensional analyses assuming perfect contact

In the case of three-dimensional flow (circular hole) with perfect contact between the geomembrane and the underlying soil, the flow is certainly limited by a bell-shaped phreatic surface similar to the phreatic surface of the two-dimensional flow (case (b) in Fig. 5). However, to the best of our knowledge, no analytical or numerical study is presently available for this case. Nonetheless, upper bound and lower bound solutions are available and they are expressed by close-form equations.

Lower bound solution. The equation related to vertical flow (similar to the two-dimensional case (c) in Fig. 5) gives a lower bound for the leakage rate and is obtained by writing Darcy's equation for a cylindrical domain:

$$Q = k_s a(h_w + H_s)/H_s \quad (27)$$

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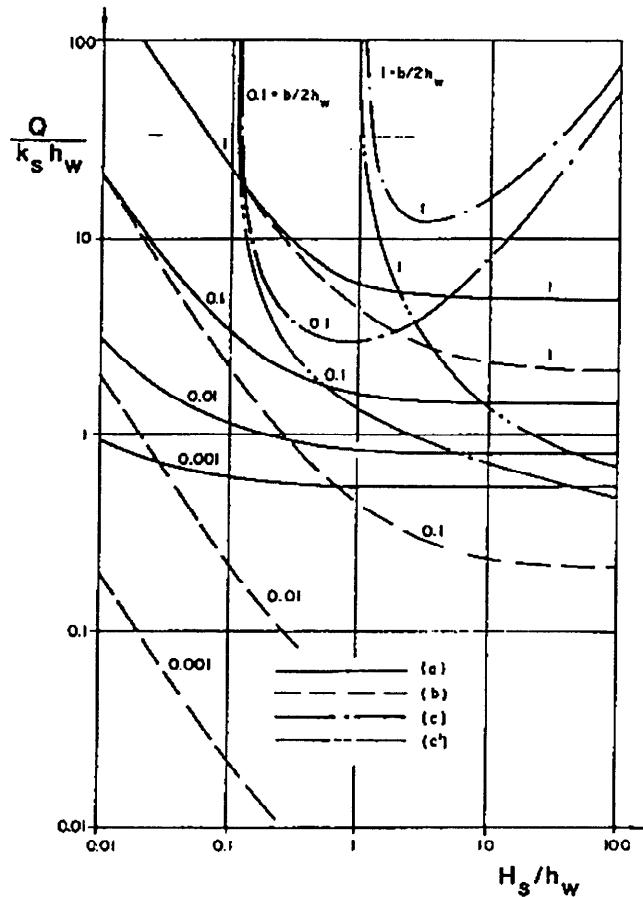


Fig. 9. Comparison of various assumptions regarding leakage rates through composite liners. The curves give leakage rates through a composite liner due to a slot of width b in the geomembrane (two-dimensional case), assuming perfect contact between the geomembrane and the low-permeability soil. Calculations were made with several assumptions regarding flow: (a) correct solution; (b) vertical flow; (c) radial flow using eqn (25); and (c') radial flow using eqn (26). Correct solution, vertical flow, and radial flow are illustrated in Fig. 5. (Adapted from Faure.¹⁸)

where: Q = lower bound of the leakage rate; k_s = hydraulic conductivity of the low-permeability soil; a = surface area of the hole in the geomembrane ($a = \pi d^2/4$ if the hole is circular); d = diameter of the circular hole; h_w = liquid depth on top of the geomembrane; and H_s = thickness of the low permeability soil. Basic SI units are: Q (m^3/s), k_s (m/s), a (m^2), d (m), h_w (m), and H_s (m).

Upper bound solution. The equation related to the three-dimensional radial flow (similar to the two-dimensional case (d) in Fig. 5) is obtained by integrating Darcy's equation for a spherical domain:

$$Q = \pi k_s (h_w + H_s) d / (1 - 0.5d/H_s) \quad (28)$$

where the notation is the same as above.

By analogy with the corresponding equation for the two-dimensional case (eqn (25)), it may be assumed that eqn (28) provides an upper bound solution for the leakage rate when the geomembrane and the underlying soil are in perfect contact. It can also be assumed that eqn (28) leads to absurd results for some cases, like eqn (25). Therefore, eqn (28) is simply used as a preliminary step to obtain a better solution, as indicated below.

First approximate solution. As discussed for eqn (25) in the two-dimensional case, eqn (28) can be arbitrarily transformed by replacing $h_w + H_s$ by h_w , which results in:

$$Q = \pi k_s h_w d / (1 - 0.5d/H_s) \quad (29)$$

It is possible that this equation gives an approximate value of the leakage rate when d/H_s is small (like eqn (26) for the two-dimensional case). In most practical cases, d/H_s is small since typical geomembrane defects are of the order of 1–10 mm (0.04–0.4 in) in diameter, whereas the low-permeability soil component of composite liners is often of the order of 1 m (3 ft) thick. It is interesting to note that eqn (29) tends toward a very simple limit when d/H_s tends toward zero:

$$Q = \pi k_s h_w d \quad (30)$$

where the notation is the same as for eqn (27).

Due to the lack of a more rigorous solution, eqn (30) will be used as an approximation for the leakage rate when there is perfect contact between the geomembrane and underlying low-permeability soil.

Second approximate solution. Another approach for evaluating leakage rates in the three-dimensional case is to use the chart established by Faure for the two-dimensional case (Fig. 8) and modify eqn (23) by replacing in Q^* (which is equal to Q/length) the length of the slot by the perimeter πd of the circular hole (and not half the perimeter, nor the diameter of the hole, as one may be tempted to do):

$$Q = \pi C_F k_s (H_s + h_w) d \quad (31)$$

where: Q = approximate value of the leakage rate; C_F = dimensionless coefficient given by Faure's chart (Fig. 8); k_s = hydraulic conductivity of the low-permeability soil; H_s = thickness of the low-permeability soil layer; h_w = liquid depth on top of the geomembrane; and d = hole

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diameter. Basic SI units are: Q (m^3/s), k_s (m/s), H_s (m), h_w (m), and d (m); C_F is dimensionless.

3.2.4 Three-dimensional analyses with interface flow

Analytical studies of leakage through composite liners due to geomembrane defects considering interface flow were published by Fukuoka²¹ and Brown *et al.*⁹ Several of the equations presented in Section 3.2.4 are adapted from these publications.

Flow mechanism. As discussed in Section 3.1.2, a fraction of the liquid that has passed through a geomembrane defect will flow laterally between the geomembrane and the soil, unless there is perfect contact between the geomembrane and the underlying soil. This 'interface flow' covers an area called the wetted area. In the case considered here, the hole in the geomembrane is assumed to be circular and the interface flow is assumed to be radial; therefore, the wetted area is circular. The interface flow takes place in the (usually small) space between the geomembrane and the underlying soil.

It is interesting to consider the case where there is a geotextile between the geomembrane and the underlying soil. Strictly speaking, this three-component liner does not meet the definition of a composite liner given in Section 1.3.2. However, results of experiments described subsequently have shown that, in some cases, the presence of a geotextile layer at the interface improves the performance of a composite liner. Therefore, it is useful to consider the case of a liner comprised of a geomembrane upper component, a geotextile middle component, and a low-permeability soil lower component. (It should be made clear that this geotextile is not a drainage layer and should not be connected to any kind of outlet, such as a pipe, sump or manhole.) If there is a geotextile between the geomembrane and the underlying low-permeability soil, the interface flow takes place partially within the geotextile and partially within the spaces between the geotextile and the geomembrane and between the geotextile and the soil.

Hydraulic head. Flow through composite liners is normally slow. Therefore, one may assume that there is no significant head loss when liquid passes through a geomembrane defect. Consequently, the hydraulic head acting on top of the low-permeability soil, just below a geomembrane defect, can be assumed to be equal to the depth of liquid on top of the geomembrane, h_w . (This implies that the top surface of the low-permeability soil layer is used as the datum for the hydraulic head and geomembrane thickness is neglected.) The hydraulic head decreases from h_w at the edge of the circular defect to zero at the periphery of the wetted area. This is illustrated by Fig. 10 in the case of a circular hole and radial interface flow.

The actual shape of the curve of the hydraulic head acting on top of the

low-permeability soil, h , as a function of the radius, r , can be given by a study of the interface flow.

Interface flow. The interface flow is the lateral flow in the space between the geomembrane and the soil. Hereafter we assume that the space between the geomembrane and the soil is uniform (e.g. an empty space of uniform thickness between the geomembrane and the soil, or a geotextile of uniform thickness and permeability between the geomembrane and the soil). We also assume that the geomembrane hole is circular and the

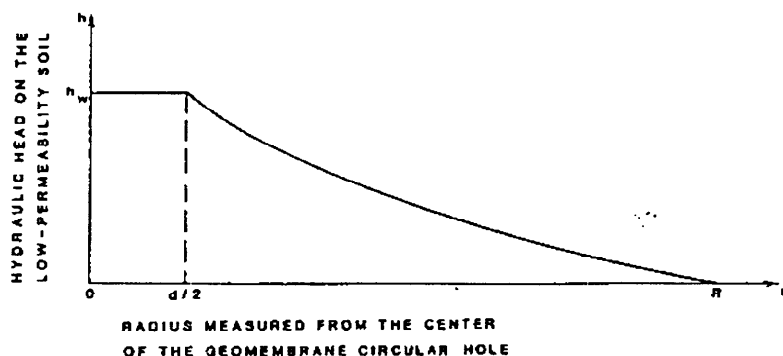


Fig. 10. Hydraulic head on top of the low-permeability soil underlying the geomembrane. The horizontal reference level for the hydraulic head is the upper surface of the low-permeability soil layer. The origin 0 of the radial axis is on the soil surface, below the center of the circular hole of the geomembrane. Legend: r = any radial distance measured from the origin 0; d = diameter of the geomembrane circular hole; R = radius of the wetted area; and h_w = depth of water on the geomembrane. (Note: For $0 \leq r \leq d/2$, the hydraulic head is actually $h = h_w + T_g + s$, where T_g = geomembrane thickness, and s = spacing between geomembrane and soil. T_g and s are very small and are neglected.)

interface flow is radial. The uniform medium between the geomembrane and the soil can be characterized by its hydraulic transmissivity θ .

If the medium is a geotextile, its hydraulic transmissivity is defined by:

$$\theta = k_p s \quad (32)$$

where: θ = geotextile hydraulic transmissivity; k_p = geotextile in-plane hydraulic conductivity; and s = spacing between the geomembrane and the underlying soil (i.e. geotextile thickness). Basic SI units are: θ (m^2/s), k_p (m/s), and s (m).

If the medium is an empty space, its hydraulic transmissivity is expressed by the following equation, adapted from Brown *et al.*,⁹ which was obtained

by applying Newton's viscosity law to flow between two smooth, parallel plates:

$$\theta = \frac{\rho g s^3}{12\eta} \quad (33)$$

where: θ = hydraulic transmissivity of the empty space; ρ = density of the liquid; g = acceleration due to gravity; s = spacing between the geomembrane and the soil; and η = viscosity of the liquid. Basic SI units are: θ (m^2/s), ρ (kg/m^3), g (m/s^2), s (m), and η ($\text{kg}/(\text{m}\cdot\text{s})$).

For example, using the density ($\rho = 1000 \text{ kg}/\text{m}^3$) and the viscosity ($\eta = 10^{-3} \text{ kg}/(\text{m}\cdot\text{s})$) of water at 20°C , eqn (33) shows that a spacing $s = 1 \text{ mm}$ is equivalent to a hydraulic transmissivity of $8 \times 10^{-4} \text{ m}^2/\text{s}$, and a spacing $s = 0.1 \text{ mm}$ is equivalent to a hydraulic transmissivity of $8 \times 10^{-7} \text{ m}^2/\text{s}$. In comparison, a typical needlepunched nonwoven geotextile with a thickness of 3 mm and a hydraulic conductivity of $1 \times 10^{-3} \text{ m/s}$ has a hydraulic transmissivity of $3 \times 10^{-6} \text{ m}^2/\text{s}$.

The flow rate related to interface flow can be expressed using Darcy's equation:

$$Q_i = k_i A = \theta i B \quad (34)$$

where: Q_i = interface flow rate; k = hydraulic conductivity of the flow medium; i = hydraulic gradient; A = cross-sectional area of the flow; θ = hydraulic transmissivity of the flow medium; and B = width of the flow. Basic SI units are: Q_i (m^3/s), k (m/s), A (m^2), θ (m^2/s), and B (m); i is dimensionless.

In the case of the considered radial flow:

$$i = -dh/dr \quad (35)$$

$$B = 2\pi r \quad (36)$$

Hence, with $Q_i = Q_r$ in eqn (34):

$$Q_r = -2\pi r \theta dh/dr \quad (37)$$

where: Q_r = interface radial flow rate at radius r ; and h = hydraulic head acting on top of the low-permeability soil.

Flow through the soil. The slope of the flow lines through the soil is not known. For the sake of simplicity, flow through the soil layer is assumed to be vertical. Darcy's equation related to flow through the soil can then be written as follows:

$$Q_s = k_s i_s A_s \quad (38)$$

where: Q_s = flow through the soil; i_s = vertical hydraulic gradient

through the soil; and A_s = cross-sectional area of the considered flow. Basic SI units are: Q_s (m^3/s), k_s (m/s), and A_s (m^2); i_s is dimensionless.

In the case considered, the vertical hydraulic gradient in the soil, i_s , varies radially because the hydraulic head acting on top of the soil, h , varies radially:

$$i_s = \frac{h + H_s}{H_s} \quad (39)$$

where: H_s = thickness of the soil layer; and h is a function of r as shown in Fig. 10.

If we consider an annular region between radii r and $r + dr$:

$$A_s = 2\pi r dr \quad (40)$$

Consequently the flow rate in the soil can be expressed as follows:

$$dQ_s = 2\pi r k_s \frac{h + H_s}{H_s} dr \quad (41)$$

Flow differential equation. The principle of conservation of mass applied to the differential element in Fig. 11 dictates that:

$$dQ_s + dQ_r = 0 \quad (42)$$

dQ_r can be derived from eqn (37):

$$dQ_r = -2\pi r \theta \left(\frac{1}{r} \frac{dh}{dr} + \frac{d^2 h}{dr^2} \right) dr \quad (43)$$

Combining eqns (41), (42) and (43) leads to:

$$\frac{1}{r} \frac{dh}{dr} + \frac{d^2 h}{dr^2} = \frac{k_s}{\theta} \left(1 + \frac{h}{H_s} \right) \quad (44)$$

Equation (44) was written by Fukuoka²¹ for the case of interface flow in a geotextile (i.e. θ given by eqn (32)), and by Brown *et al.*⁹ for the case of interface flow in an empty space between the geomembrane and the soil

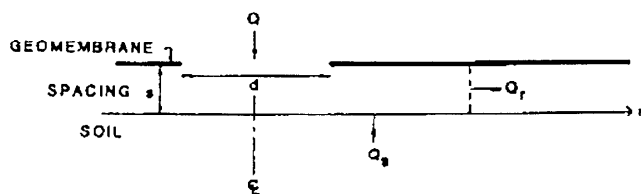


Fig. 11. Relationship between radial interface flow, Q_r , and flow into the soil, Q_s .

(i.e. θ given by eqn (33)). As shown by Brown *et al.*,⁹ this differential equation can be solved using Bessel functions.

Special case. In order to simplify eqn (44), Brown *et al.*⁹ assumed that the hydraulic gradient for the vertical flow in soil is one. According to eqn (39), this simplifying assumption is acceptable if the hydraulic head acting on the low-permeability soil is negligible compared to the thickness of the low-permeability soil. Values of hydraulic heads acting on liners discussed in Section 1.3.6 show that the above simplifying assumption is:

- always acceptable for bottom liners;
- unconservative for liquid impoundment top liners; and
- acceptable in most cases of top liners for landfills.

Using $i_z = 1$ and applying the principle of conservation of mass to the flow between the geomembrane hole and the boundary of the differential element in Fig. 11 results in:

$$Q = Q_r + Q_s \quad (45)$$

where Q_r is given by eqn (37), and:

$$Q = \pi R^2 k_s \quad (46)$$

$$Q_s = \pi r^2 k_s \quad (47)$$

where: Q = leakage rate; Q_r = interface radial flow rate at radius r ; Q_s = flow into the soil within radius r ; R = radius of wetted area; and k_s = hydraulic conductivity of the soil. Basic SI units are: Q , Q_r and Q_s (m^3/s), r and R (m), and k_s (m/s).

Combining eqns (37), (45), (46) and (47) leads to:

$$\frac{dh}{dr} = \frac{k_s}{2\theta} \left(r - \frac{R^2}{r} \right) \quad (48)$$

Integrating this differential equation leads to the equation of the h - r curve:

$$h = \frac{R^2 k_s}{4\theta} \left[2 \ln \frac{R}{r} + \left(\frac{r}{R} \right)^2 - 1 \right] \quad (49)$$

and the relationship between the depth of liquid, h_w and the radius of the wetted area:

$$h_w = \frac{R^2 k_s}{4\theta} \left[2 \ln \frac{2R}{d} + \left(\frac{d}{2R} \right)^2 - 1 \right] \quad (50)$$

where: h_w = depth of liquid on the geomembrane; R = radius of the wetted area; k_s = hydraulic conductivity of the low-permeability soil;

θ = hydraulic transmissivity of the space between the geomembrane and the low-permeability soil (given by eqn (32) if there is a geotextile between the geomembrane and the low-permeability soil and by eqn (33) if the space between the geomembrane and the low-permeability soil is empty); and d = diameter of the geomembrane hole. (Note that eqn (49) satisfies eqn (44) with $h = 0$, i.e. $i_s = 1$.)

Equation (50) gives the radius of the wetted area if the hydraulic transmissivity of the space between the geomembrane and the low-permeability soil is known and if the depth of liquid on top of the geomembrane is known. When the radius of the wetted area, R , is determined, the leakage rate, Q , can be determined using eqn (46).

The most uncertain step in using the above equations is the selection of the hydraulic transmissivity value, θ . If there is a geotextile between the geomembrane and the underlying soil, and if the geotextile is in close contact with the overlying geomembrane and the underlying soil, the hydraulic transmissivity, θ , can be obtained by conducting hydraulic transmissivity tests on samples of the geotextile subjected to a normal stress equal to the normal stress in the field. If there is no geotextile between the geomembrane and the soil, the hydraulic transmissivity to use in eqn (50) is given by eqn (33). To use eqn (33), it is necessary to estimate the spacing, s , between the geomembrane and the underlying soil. This spacing depends on the rugosity of the soil surface, the stiffness of the geomembrane, and the magnitude of the normal stress that tends to press the geomembrane against the soil. Recommended values for the spacing, s , were given by Brown *et al.*⁹ on the basis of model tests, and can be found hereafter in Section 3.3.2. Using these recommended values, Brown *et al.*⁹ established charts giving the leakage rate and the radius of the wetted area as a function of the geomembrane hole surface area, the soil hydraulic conductivity and the depth of water on the geomembrane. To summarize results presented in these charts and to extrapolate or interpolate them, we propose the following empirical equations:

$$Q = 0.7 a^{0.1} k_s^{0.88} h_w \quad (51)$$

$$R = 0.5 a^{0.05} k_s^{-0.06} h_w^{0.5} \quad (52)$$

These equations are only valid with the units indicated: Q = leakage rate (m^3/s); a = geomembrane hole area (m^2); k_s = hydraulic conductivity of low-permeability soil (m/s); h_w = liquid depth on the geomembrane (m); and R = radius of wetted area between geomembrane and soil (m). Equations (51) and (52) result from a combination of the theoretical analysis presented above and experimental data presented in Section 3.3.2. Note that, as discussed prior to eqn (45), eqns (51) and (52) are

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based upon assumptions that are valid only if the liquid depth on the geomembrane is less than the thickness of the low-permeability soil layer.

Upper bound solution. An upper bound of the flow rate and the radius of the wetted area occurs when the spacing s between the geomembrane and the underlying soil is so large that the flow rate through a geomembrane defect is given by eqn (22), that is the Bernoulli's equation related to free flow through an orifice. By combining eqns (22) and (46), it appears that, if the spacing between the geomembrane and the soil is large enough to ensure free flow, the radius of the wetted area is given by:

$$\pi R^2 k_s = 0.6 a \sqrt{2gh_w} \quad (53)$$

hence:

$$R = 0.44 a^{0.5} (2gh_w)^{0.25} k_s^{-0.5} \quad (54)$$

and, in the case of a circular hole:

$$R = 0.39 d (2gh_w)^{0.25} k_s^{-0.5} \quad (55)$$

where: R = radius of the wetted area; a = geomembrane hole area; d = hole diameter; g = acceleration due to gravity; h_w = liquid depth on the geomembrane; and k_s = hydraulic conductivity of the low-permeability soil underlying the geomembrane. Basic SI units are: R (m), a (m²), d (m), g (m/s²), h_w (m), and k_s (m/s).

Note that the radii given by eqns (52), (54) and (55) correspond to cases where there is no overlapping between wetted areas related to different geomembrane holes.

3.3 Laboratory model tests

3.3.1 Introduction

Tests to evaluate leakage rates through composite liners due to geomembrane holes were conducted by Fukuoka^{21,22} and Brown *et al.*⁹

In both studies, tests were conducted with a geomembrane having a circular hole, and various hole diameters were tested. Additional tests by Brown *et al.* included geomembrane flaws that were not circular such as slits or seam defects. The tests were intended to be full-scale models since hole size, geomembrane thickness and (approximately) soil layer thickness were similar to typical field values. However, the permeameters used had a limited diameter (e.g. 0.6 m (2 ft) for Brown *et al.* and 1.5 m (5 ft) for Fukuoka) and the extent of lateral flow between the geomembrane and soil was limited by the walls of the permeameter.

In the tests conducted by Brown *et al.*, the geomembrane was always covered by 0.15 m (6 in) of gravel to ensure contact between geomem-

brane and soil, and, in some tests, an additional load up to 160 kPa (3340 psf) (equivalent to 10 m (30 ft) of soil) was applied to evaluate the effect of overburden pressure. In many of the tests conducted by Fukuoka, the geomembrane was not covered, and the only load applied on the geomembrane was the water pressure.

Water depths on the geomembrane in the Brown *et al.* tests were up to 1 m (40 in). In the Fukuoka tests, water pressures equivalent to water depths up to 40 m (130 ft) were used. Tests by Brown *et al.* were conducted for landfill applications while Fukuoka was working on the design of a large dam and reservoir.

Fukuoka used only a PVC geomembrane, while Brown *et al.* considered a variety of geomembranes: HDPE, PVC, CSPE and EPDM, with various thicknesses. Some of the tests conducted by Fukuoka and by Brown *et al.* included a geotextile between the geomembrane and the soil.

Tests by Fukuoka as well as tests by Brown *et al.* showed that there is significant flow between the geomembrane and the soil, with or without geotextile.

3.3.2 Review of tests by Brown *et al.*

These tests are presented in a report by Brown *et al.*⁹

Description of the tests. Tests were conducted in a 0.6 m (24 in) diameter permeameter. Geomembrane hole diameters ranged between 0.8 mm (0.03 in) and 13 mm (0.5 in), and noncircular holes such as slits and seam defects were considered.

The geomembranes and their thicknesses were: HDPE, 0.75–2.5 mm (30–100 mils); PVC, 0.5–0.75 mm (20–30 mils); CSPE, 0.9–1.15 mm (36–45 mils); and EPDM, 0.75 mm (30 mils). In some tests, geotextiles were included between the geomembrane and the soil. The geotextiles were needlepunched nonwoven materials with masses per unit area of 250–350 g/m² (7–10 oz/yd²) and thicknesses (under no load) of the order of 2.5–4 mm (0.10–0.16 in).

The soils used were a silty sand ($k_s = 2 \times 10^{-6}$ m/s), and a clayey silt ($k_s = 2 \times 10^{-8}$ m/s).

Approach. The diameter of the permeameter used by Brown *et al.* was small (0.6 m (24 in)) and lateral flow could not extend beyond a radius of 0.3 m (12 in) as it would have in most cases without the limitation imposed by the permeameter walls. This fact was recognized by Brown *et al.* who did not use their tests to evaluate the leakage rate directly. Instead, they conducted calculations similar to those presented in Section 3.2.4 (but taking into account the presence of the permeameter walls) to derive the value of the spacing between the geomembrane and soil from the test results. The value of the spacing thus obtained can be used in eqn (33) to

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calculate the value of θ required in eqn (50) to determine the radius of the wetted area and, therefore, the leakage rate in situations where lateral flow is not impeded by permeameter walls. The spacing values determined by Brown *et al.* are as follows:

- 0.02 mm for clayey silt regardless of geomembrane type
- 0.08 mm for silty sand and flexible geomembranes (PVC)
- 0.15 mm for silty sand and stiff geomembranes (HDPE)

The spacing between the geomembrane and the soil, and, therefore, the leakage rate, appears to increase if the geomembrane stiffness increases (at least in the case of the more permeable soil). It also appears that spacing increases if the soil is coarse, which is illustrated by the following:

- 0.02 mm = d_{10} of the clayey silt used in the tests
- 0.08 mm = d_{15} of the silty sand used in the tests

The above spacing values are related to the case of a geomembrane with 15 cm (6 in) of gravel overburden. This is a very low overburden pressure in comparison to field conditions. As a result, one may conclude that the above spacing values are larger than under field conditions. Such a conclusion is not necessarily correct since, in the field, geomembranes have wrinkles and subgrade preparation is not as good as in the tests.

Following is our review of the influence of various parameters on the test results of Brown *et al.*

Influence of geotextile between geomembrane and underlying soil. On the clayey silt ($k_s = 2 \times 10^{-8}$ m/s (2×10^{-6} cm/s)), the geotextile does not change the leakage rate. On the silty sand ($k_s = 2 \times 10^{-6}$ m/s (2×10^{-4} cm/s)), the geotextile seems to reduce slightly the leakage rate.

Effect of overburden pressures. When a compressive stress of 160 kPa (3340 psf) (equivalent to 10 m (30 ft) of soil) is applied on a 0.75 mm (30 mil) thick HDPE geomembrane placed on a soil with a hydraulic conductivity of 2×10^{-6} m/s (2×10^{-4} cm/s), the leakage rate is divided by 200 and the calculated theoretical spacing between geomembrane and soil is divided by 10, as compared to the case where the overburden pressure was 1.5 kPa (30 psf). (There are no results for the soil with a hydraulic conductivity of 2×10^{-8} m/s (2×10^{-6} cm/s).) Based on this limited result, the effect of overburden pressure on the leakage rate appears to be significant.

Effect of flaw shape. Erratic results were obtained with geomembrane slits and seam defects on the soil with $k_s = 2 \times 10^{-6}$ m/s (2×10^{-4} cm/s), and it was difficult to compare slits, seams and circular holes with the 2×10^{-8} m/s (2×10^{-6} cm/s) soil because for that soil there is a large lateral flow and permeameter walls disturbed the flow. Therefore, in our

opinion, it is not possible to draw conclusions regarding the effect of flaw shape.

Conclusions from Brown et al.'s tests. In order to extrapolate to field conditions, Brown *et al.* make the recommendations in Table 8 regarding the values of the spacing between geomembrane and soil to be used in the equations presented in Section 3.2.4 to evaluate leakage rate and radius of wetted area in actual field conditions where lateral extension of flow is not impeded by wall permeameter.

TABLE 8

Soil hydraulic conductivity, k_s (m/s)	Geomembrane-soil spacing, s (mm)
10^{-6}	0.15
10^{-7}	0.08
10^{-8}	0.04
10^{-9}	0.02

These values are larger than the calculated spacing values previously given in the discussion of the approach. Also, these spacing values are for the case when there is little or no overburden (e.g. 15 cm (6 in) of gravel), and smaller spacing values would have been obtained with overburden of the order of typical field values. Therefore, the above spacing values are large in the case of laboratory conditions. We will assume that these values can be used in the case of excellent field conditions (defined subsequently), based on the following rationale: (i) on one hand, in the field, overburden stresses, which tend to decrease the spacing, are larger than the 15 cm (6 in) of gravel used as overburden in the tests; and (ii), on the other hand, for a given overburden stress, spacing between the geomembrane and the soil is larger in the field than in the laboratory tests since, even under excellent field conditions, geomembranes always have wrinkles and soil preparation is never as good as in the tests.

The above spacing values were used by Brown *et al.*⁹ to establish their charts (not reproduced here), which we used to establish eqns (51) and (52). Therefore, values given by these equations can be assumed to represent excellent field conditions.

3.3.3 Review of tests by Fukuoka

These tests are described by Fukuoka.^{21,22} They were conducted for the design of the lining system for a dam and a reservoir with a maximum water head of 40 m (130 ft).

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